The black mark (▬) indicates Starting Points.
SCT: Standard Completion Time (Min./Sheet)
This is the time in which the student should complete each worksheet, including time taken for corrections.

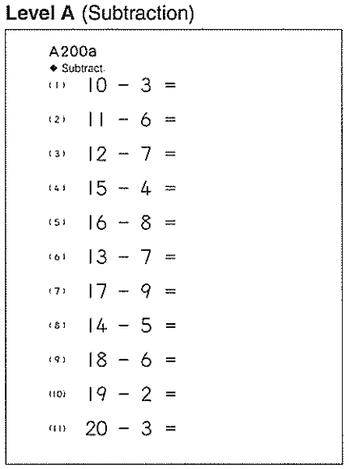
TABLE OF LEARNING MATERIALS I-2
MATHEMATICS (A-F)

KUMON

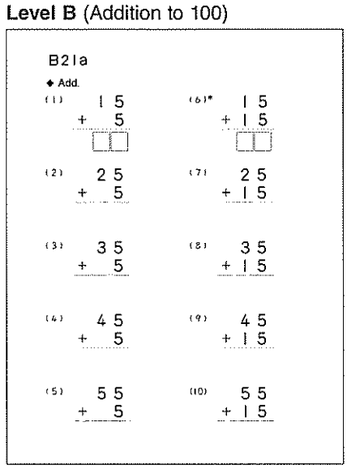
October 2016

	A	SCT	B	SCT	C	SCT	D	SCT	E	SCT	F	SCT
1- 10	Addition 1 (Review up to 2A)	1–2	Addition (Review up to A)	1–2	Review up to B	2–3	Review up to C	2–3	Review up to D	2–3	Review up to E 1	3–5
11- 20	Addition 2 (Up to sum of 12)	1–2	Addition to 100 Part 1	1–2	Multiplication up to 3	2–3	Multiplication: 2 Digits × 2 Digits 1	2–4	Addition of Fractions 1	2–3	Review up to E 2	3–5
21- 30	Addition 3 (Up to sum of 15)	1–2	Addition to 100 Part 2	2–3	Multiplication up to 5	2–3	Multiplication: 2 Digits × 2 Digits 2	2–4	Addition of Fractions 2	3–4	Multiplication and Division of 3 Fractions	4–6
31- 40	Addition 4 (Up to sum of 18)	1–2	Addition to 100 Part 3	2–3	Multiplication up to 7	2–3	Multiplication: 2 Digits × 2 Digits 3	3–4	Addition of Fractions 3	3–4	Addition of 3 Fractions 1	3–5
41- 50	Addition 5 (Up to sum of 20)	1–2	Addition of 2-Digit Numbers 1	2–3	Multiplication up to 9	2–3	Multiplication: 3 Digits × 2 Digits	3–5	Addition of Fractions 4	3–4	Addition of 3 Fractions 2	4–6
51- 60	Addition 6 (Up to sum of 24)	1–2	Addition of 2-Digit Numbers 2	2–3	Multiplication: 2 Digits × 1 Digit 1	2–4	Addition and Subtraction	2–4	Addition of Fractions 5	3–5	Addition and Subtraction of 3 Fractions	4–6
61- 70	Addition 7 (Up to sum of 28)	1–2	Addition of 2-Digit Numbers 3	2–3	Multiplication: 2 Digits × 1 Digit 2	2–4	Multiplication and Division 1	3–4	Addition of Fractions 6	3–5	Order of Operations (3 Fractions) 1	3–5
71- 80	Addition 8 (Summary of addition)	2–3	Addition of 3-Digit Numbers 1	2–4	Multiplication: 2 Digits × 1 Digit 3	2–4	Multiplication and Division 2	3–4	Addition of Fractions 7	3–5	Order of Operations (3 Fractions) 2	3–5
81- 90	Subtraction 1 (Subtracting 1)	1–2	Addition of 3-Digit Numbers 2	2–4	Multiplication: 2 Digits × 1 Digit 4	2–4	Division by 2-Digit Numbers 1	3–4	Addition of Fractions 8	3–5	Order of Operations (3 Fractions) 3	3–5
91-100	Subtraction 2 (Subtracting 2)	1–2	Addition of 3-Digit Numbers 3	3–5	Multiplication: 2 Digits × 1 Digit 5	2–4	Division by 2-Digit Numbers 2	3–5	Addition of Fractions 9	4–6	Order of Operations (3-or-More Fractions) 1	4–6
101-110	Subtraction 3 (Subtracting 3)	1–2	Subtraction 1 (Review up to A)	1–2	Multiplication: 3 or 4 Digits × 1 Digit	3–5	Division by 2-Digit Numbers 3	3–5	Subtraction of Fractions 1	3–4	Order of Operations (3-or-More Fractions) 2	4–6
111-120	Subtraction 4 (Subtracting up to 3)	1–2	Subtraction 2 (Review up to A)	2–3	Introduction to Division	2–3	Division by 2-Digit Numbers 4	3–5	Subtraction of Fractions 2	3–5	Order of Operations (3-or-More Fractions) 3	4–6
121-130	Subtraction 5 (Subtracting up to 5)	1–2	Subtraction of 2-Digit Numbers 1	2–3	Division with Remainders 1	2–3	Division by 2-Digit Numbers 5	3–5	Subtraction of Fractions 3	3–5	Order of Operations (3-or-More Fractions) 4	4–6
131-140	Subtraction 6 (From numbers up to 10)	1–2	Subtraction of 2-Digit Numbers 2	2–3	Division with Remainders 2	2–3	Division: Quotients of 2-or-More Digits 1	3–5	Addition and Subtraction of Fractions	4–6	Fractions and Decimals 1	3–5
141-150	Subtraction 7 (From numbers up to 11)	1–2	Subtraction of 2-Digit Numbers 3	2–3	Division with Remainders 3	2–3	Division: Quotients of 2-or-More Digits 2	4–6	Multiplication of Fractions 1	3–4	Fractions and Decimals 2	4–6
151-160	Subtraction 8 (From numbers up to 12)	1–2	Addition and Subtraction of 2-Digit Numbers	2–4	Division with Remainders 4	2–3	Fractions	3–5	Multiplication of Fractions 2	3–5	Fractions and Decimals 3	4–6
161-170	Subtraction 9 (From numbers up to 14)	1–2	Subtraction of 3-Digit Numbers 1	2–3	Division: 2 Digits ÷ 1 Digit 1	2–3	Reduction 1	2–3	Division of Fractions	3–5	Solving Word Problems 1	4–6
171-180	Subtraction 10 (From numbers up to 16)	1–2	Subtraction of 3-Digit Numbers 2	2–4	Division: 2 Digits ÷ 1 Digit 2	2–3	Reduction 2	2–3	Multiplication and Division of Fractions	3–5	Solving Word Problems 2	5–7
181-190	Subtraction 11 (From numbers up to 20)	1–2	Subtraction of 3-Digit Numbers 3	2–4	Division: 3 Digits ÷ 1 Digit 1	3–4	Reduction 3	2–4	Four Operations of 2 Fractions 1	3–5	Decimals 1	3–5
191-200	Subtraction 12 (Summary of subtraction)	2–3	Subtraction of 3-Digit Numbers 4	3–5	Division: 3 Digits ÷ 1 Digit 2	3–5	Reduction 4	2–4	Four Operations of 2 Fractions 2	3–5	Decimals 2	3–5

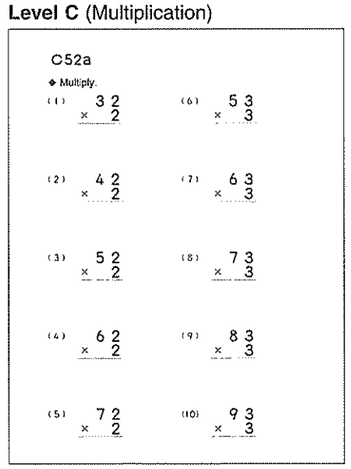
Level A (Subtraction)



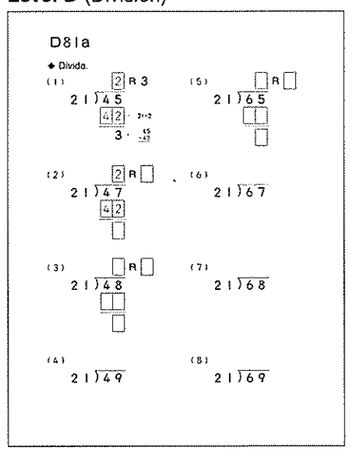
Level B (Addition to 100)



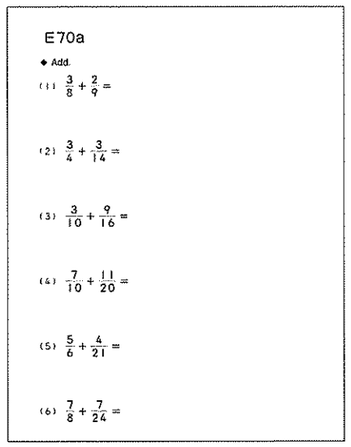
Level C (Multiplication)



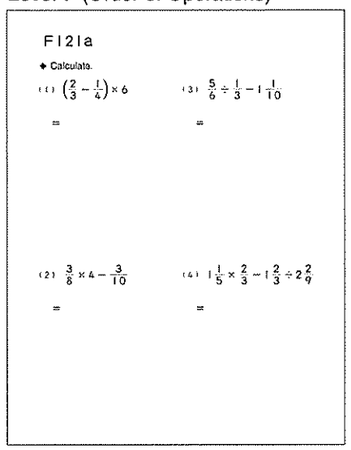
Level D (Division)



Level E (Addition of Fractions)



Level F (Order of Operations)



The black mark (▬) indicates Starting Points.
SCT: Standard Completion Time (Min./Sheet)
This is the time in which the student should complete each worksheet, including time taken for corrections.
(SCT is not used in Levels 6A-5A and ZI-ZII.)

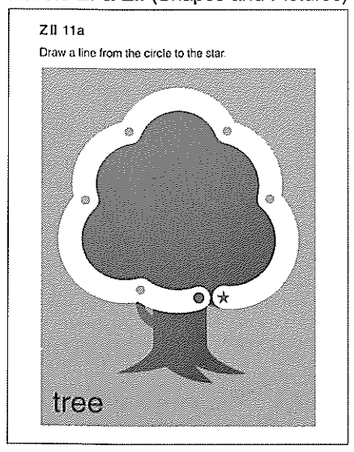
TABLE OF LEARNING MATERIALS I-1
MATHEMATICS (6A-2A) / PENCIL SKILLS PROGRAMME (ZI-ZII)

KUMON

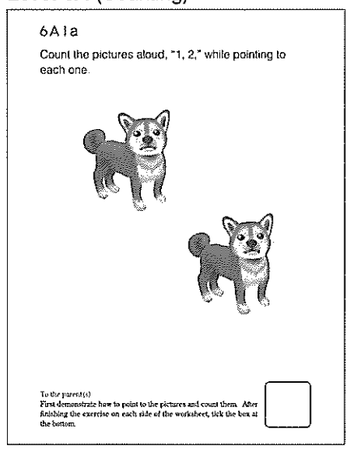
December 2012

	Pencil Skills Programme		6A	5A	4A	SCT	3A	SCT	2A	SCT
ZI	1- 10	Colouring 1	1- 10	Counting (Up to 5) 1	Number Reading Exercises (Up to 30) 1	Number Tracing Exercises 1	0.5–2	Numbers up to 100 Part 1	0.5–2	Review up to 3A	1–2
	11- 20	Colouring 2	11- 20	Counting (Up to 5) 2	Number Reading Exercises (Up to 30) 2	Number Tracing Exercises 2	0.5–2	Numbers up to 100 Part 2	0.5–2	Adding 4 Part 1 (Up to 12+4)	1–2
	21- 30	Straight Lines 1	21- 30	Counting (Up to 5) 3	Number Reading Exercises (Up to 30) 3	Number Tracing Exercises 3	0.5–2	Numbers up to 100 Part 3	0.5–2	Adding 4 Part 2 (Up to 16+4)	1–2
	31- 40	Straight Lines 2	31- 40	Counting (Up to 10) 1	Number Reading Exercises (Up to 30) 4	Number Tracing Exercises 4	0.5–2	Numbers up to 100 Part 4	0.5–2	Adding 5 Part 1 (Up to 12+5)	1–2
	41- 50	Straight Lines 3	41- 50	Counting (Up to 10) 2	Number Reading Exercises (Up to 30) 5	Number Writing Exercises up to 10 Part 1	0.5–2	Numbers up to 100 Part 5	0.5–2	Adding 5 Part 2 (Up to 15+5)	1–2
	51- 60	Curved Lines 1	51- 60	Counting (Up to 10) 3	Number Reading Exercises (Up to 30) 6	Number Writing Exercises up to 10 Part 2	0.5–2	Numbers up to 100 Part 6	0.5–2	Adding up to 5 Part 1	1–2
	61- 70	Curved Lines 2	61- 70	Counting (Up to 10) 4	Number Reading Exercises (Up to 30) 7	Number Writing Exercises up to 10 Part 3	0.5–2	Numbers up to 120	1–2	Adding up to 5 Part 2	1–2
	71- 80	Curved Lines 3	71- 80	Counting (Up to 10) 5	Number Reading Exercises (Up to 30) 8	Number Writing Exercises up to 10 Part 4	0.5–2	Adding 1 Part 1 (Up to 12+1)	1–2	Adding 6 Part 1 (Up to 12+6)	1–2
	81- 90	Curved Lines 4	81- 90	Counting (Up to 10) 6	Number Reading Exercises (Up to 30) 9	Number Writing Exercises up to 10 Part 5	0.5–2	Adding 1 Part 2 (Up to 18+1)	1–2	Adding 6 Part 2 (Up to 14+6)	1–2
	91-100	Curved Lines 5	91-100	Counting (Up to 10) 7	Number Reading Exercises (Up to 30) 10	Number Writing Exercises up to 10 Part 6	0.5–2	Adding 1 Part 3 (Up to 24+1)	1–2	Adding 7 Part 1 (Up to 11+7)	1–2
ZII	1- 10	Shapes and Pictures 1	101-110	Number Reading Exercises (Up to 10) 1	Sequence of Numbers (Up to 30) 1	Number Writing Exercises up to 20 Part 1	0.5–2	Adding 1 Part 4 (Up to 30+1)	1–2	Adding 7 Part 2 (Up to 13+7)	1–2
	11- 20	Shapes and Pictures 2	111-120	Number Reading Exercises (Up to 10) 2	Sequence of Numbers (Up to 30) 2	Number Writing Exercises up to 20 Part 2	0.5–2	Adding 1 Part 5 (Up to 60+1)	1–2	Adding up to 7 Part 1	1–2
	21- 30	Shapes and Pictures 3	121-130	Number Reading Exercises (Up to 10) 3	Sequence of Numbers (Up to 30) 3	Number Writing Exercises up to 30 Part 1	0.5–2	Adding 1 Part 6 (Up to 1000+1)	1–2	Adding up to 7 Part 2	1–2
	31- 40	Shapes and Pictures (Stories) 1	131-140	Number Reading Exercises (Up to 10) 4	Sequence of Numbers (Up to 40) 1	Number Writing Exercises up to 30 Part 2	0.5–2	Adding 2 Part 1 (Up to 14+2)	1–2	Adding 8 Part 1 (Up to 11+8)	1–2
	41- 50	Shapes and Pictures (Stories) 2	141-150	Number Reading Exercises (Up to 10) 5	Sequence of Numbers (Up to 40) 2	Numbers up to 50 Part 1	0.5–2	Adding 2 Part 2 (Up to 18+2)	1–2	Adding 8 Part 2 (Up to 12+8)	1–2
	51- 60	Shapes and Pictures (Stories) 3	151-160	Number of Dots (Up to 10) 1	Sequence of Numbers (Up to 40) 3	Numbers up to 50 Part 2	0.5–2	Adding 2 Part 3 (Up to 32+2)	1–2	Adding 9 (Up to 12+9)	1–2
	61- 70	Back and Forth 1	161-170	Number of Dots (Up to 10) 2	Sequence of Numbers (Up to 50) 1	Numbers up to 50 Part 3	0.5–2	Adding 3 Part 1 (Up to 14+3)	1–2	Adding 9 and 10 (Up to 12+9 and 15+10)	1–2
	71- 80	Back and Forth 2	171-180	Number of Dots (Up to 10) 3	Sequence of Numbers (Up to 50) 2	Numbers up to 50 Part 4	0.5–2	Adding 3 Part 2 (Up to 21+3)	1–2	Adding up to 10 Part 1	1–2
	81- 90	Corners and Curves 1	181-190	Number of Dots (Up to 10) 4	Sequence of Numbers (Up to 50) 3	Numbers up to 50 Part 5	0.5–2	Adding up to 3 Part 1	1–2	Adding up to 10 Part 2	1–2
	91-100	Corners and Curves 2	191-200	Number of Dots (Up to 10) 5	Large Numbers	Numbers up to 50 Part 6	0.5–2	Adding up to 3 Part 2	1–2	Adding up to 10 Part 3	1–2

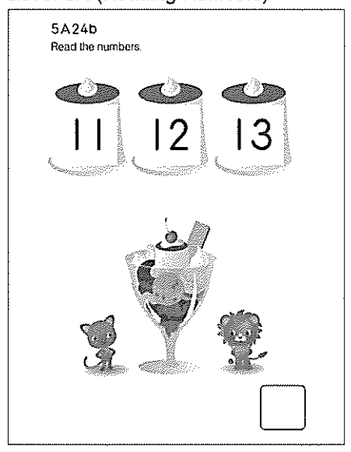
Levels ZI & ZII (Shapes and Pictures)



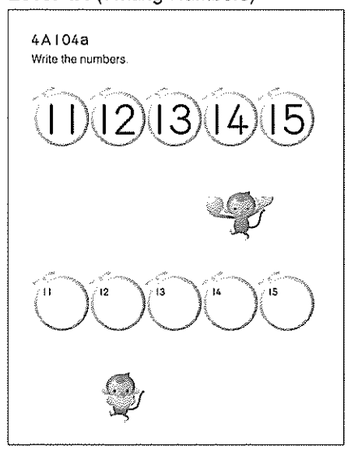
Level 6A (Counting)



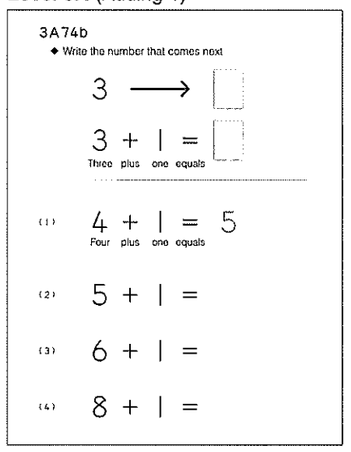
Level 5A (Reading Numbers)



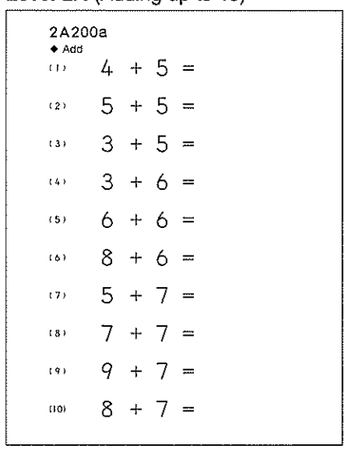
Level 4A (Writing Numbers)



Level 3A (Adding 1)



Level 2A (Adding up to 10)



Printed in Malaysia FM 1805

TABLE OF LEARNING MATERIALS II-1 (Level G to L)

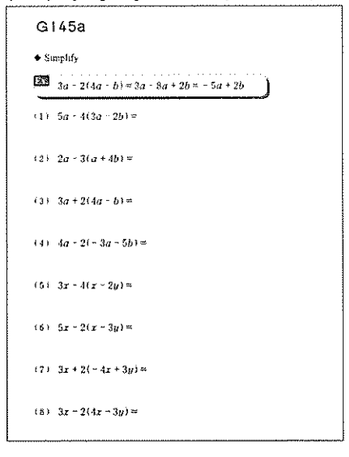
KUMON

The black mark (▬) indicates most suitable Starting Points.
SCT: Standard Completion Time (Min./Sheet)
This is the set time in which the student should complete each worksheet, including time taken for correction.

October 2016

	G	SCT	H	SCT	I	SCT	J	SCT	K	SCT	L	SCT
1- 10	▬ Review up to F 1	3–5	▬ Basics for Level H Mathematics 1	4–6	▬ Basics for Level I Mathematics	4–6	▬ Expansion of Polynomial Products	5–8	Review of Linear Functions	4–6	Logarithmic Functions	6–12
11- 20	Review up to F 2	3–5	Basics for Level H Mathematics 2	4–6	Multiplication of Polynomials	4–6	Factorisation I	5–8	Review of Quadratic Functions	5–8	Graphs of Logarithmic Functions	7–14
21- 30	Addition and Subtraction of Positive and Negative Numbers 1	2–4	Literal Equations 1	4–6	Multiplication Using Formulas	4–6	Factorisation II	5–8	Quadratic Functions and Graphs	6–12	Logarithmic Equations and Inequalities	8–16
31- 40	Addition and Subtraction of Positive and Negative Numbers 2	3–5	Literal Equations 2	4–6	▬ Factorisation 1	4–6	Factorisation III	6–10	Determining Equations of Quadratic Functions	7–14	Modulus Functions	8–16
41- 50	Addition and Subtraction of Positive and Negative Numbers 3	3–5	Simultaneous Equations in Two Variables 1	4–6	Factorisation 2	4–6	Factorisation IV	6–10	Maxima and Minima of Quadratic Functions I	7–14	Limits and Derivatives	8–16
51- 60	Addition and Subtraction of Positive and Negative Numbers 4	4–6	Simultaneous Equations in Two Variables 2	4–6	Factorisation 3	4–6	Factorisation V	7–12	Maxima and Minima of Quadratic Functions II	7–14	Tangents	12–24
61- 70	Multiplication of Positive and Negative Numbers	3–5	Simultaneous Equations in Two Variables 3	4–6	Factorisation 4	4–6	Fractional Expressions	6–10	Maxima and Minima of Quadratic Functions III	8–16	Relative Maxima and Minima I	15–30
71- 80	Division of Positive and Negative Numbers	4–6	Simultaneous Equations in Two Variables 4	4–6	Factorisation 5	4–6	Irrational Numbers I	5–8	Quadratic Functions and Equations	7–14	Relative Maxima and Minima II	15–30
81- 90	Four Operations with Positive and Negative Numbers 1	4–6	Simultaneous Equations in Two Variables 5	4–6	Square Roots 1	4–6	Irrational Numbers II	6–10	Quadratic Functions and Inequalities	7–14	Maxima and Minima I	15–30
91-100	Four Operations with Positive and Negative Numbers 2	4–6	Simultaneous Equations in Three and Four Variables 1	4–6	Square Roots 2	4–6	Quadratic Equations I	5–8	Quadratic Functions and Solutions of Quadratic Equations	8–16	Maxima and Minima II	15–30
101-110	▬ Values of Algebraic Expressions 1	4–6	Simultaneous Equations in Three and Four Variables 2	4–6	Square Roots 3	4–6	Quadratic Equations II	6–10	Higher Degree Functions	6–12	Applications to Equations and Inequalities	15–30
111-120	Values of Algebraic Expressions 2	4–6	Application of Equations	4–6	Quadratic Equations 1	4–6	Quadratic Equations and Complex Numbers	6–10	Higher Degree Equations and Inequalities	7–14	Indefinite and Definite Integrals	8–16
121-130	Simplifying Algebraic Expressions 1	3–5	Inequalities 1	4–6	Quadratic Equations 2	4–6	Discriminant	6–10	Graphs of Fractional Functions I	7–14	Definite Integrals I	12–24
131-140	Simplifying Algebraic Expressions 2	3–5	Inequalities 2	4–6	Quadratic Equations 3	4–6	Root-Coefficient Relationships	6–10	Graphs of Fractional Functions II	7–14	Definite Integrals II	12–24
141-150	Simplifying Algebraic Expressions 3	4–6	Functions and Graphs 1	4–6	Graphs of Quadratic Functions 1	4–6	Simultaneous Equations	6–10	Fractional Equations and Inequalities	8–16	Areas I	15–30
151-160	Simplifying Algebraic Expressions 4	4–6	Functions and Graphs 2	4–6	Graphs of Quadratic Functions 2	4–6	Dividing Polynomials	6–10	Graphs of Irrational Functions	7–14	Areas II	15–30
161-170	Linear Equations 1	3–5	Functions and Graphs 3	4–6	Graphs of Quadratic Functions 3	4–6	Remainder Theorem	6–10	Irrational Equations and Inequalities	8–16	Volumes	15–30
171-180	Linear Equations 2	4–6	Functions and Graphs 4	4–6	The Pythagorean Theorem 1	4–6	Factor Theorem	6–10	Exponential Functions	6–12	Velocity and Distance	15–30
181-190	Linear Equations 3	4–6	Simplifying Monomials and Polynomials 1	4–6	The Pythagorean Theorem 2	4–6	Proof of Identities	6–10	Graphs of Exponential Functions	7–14	Summary of Differentiation and Integration I	30–60
191-200	Linear Equations 4	4–6	Simplifying Monomials and Polynomials 2	4–6	The Pythagorean Theorem 3	4–6	Proof of Inequalities	7–12	Exponential Equations and Inequalities	8–16	Summary of Differentiation and Integration II	30–60

Level G
(Simplifying Algebraic Expressions)


G I 45a

◆ Simplify

Ex $3a-2(4a-b)=3a-8a+2b=-5a+2b$

(1) $5a-4(3a-2b)=$

(2) $2a-3(a+4b)=$

(3) $3a+2(4a-b)=$

(4) $4a-2(-3a-5b)=$

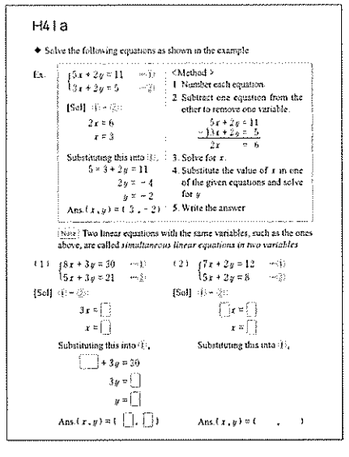
(5) $3x-4(x-2y)=$

(6) $5x-2(x-3y)=$

(7) $3x+2(-4x+3y)=$

(8) $3x-2(4x-3y)=$

Level H
(Simultaneous Equations)


H4 I a

◆ Solve the following equations as shown in the example

Ex $\begin{cases}5x+2y=11 & \cdots①\\ 3x+2y=5 & \cdots②\end{cases}$

<Method>
1. Number each equation.
2. Subtract one equation from the other to remove one variable.
3. Solve for x.
4. Substitute the value of x in one of the given equations and solve for y.
5. Write the answer

[Sol] ① − ②:
$2x=6$
$x=3$

Substituting this into ①,
$5\times3+2y=11$
$2y=-4$
$y=-2$

Ans. $(x, y)=(3, -2)$

$5x+2y=11$
$-)\ 3x+2y=5$
$2x=6$

Note: Two linear equations with the same variables, such as the ones above, are called *simultaneous linear equations in two variables*

(1) $\begin{cases}8x+3y=30 & \cdots①\\ 5x+3y=21 & \cdots②\end{cases}$

[Sol] ① − ②:
$3x=$ ☐
$x=$ ☐

Substituting this into ①,
☐ $+3y=30$
$3y=$ ☐
$y=$ ☐

Ans. $(x, y)=($ ☐, ☐ $)$

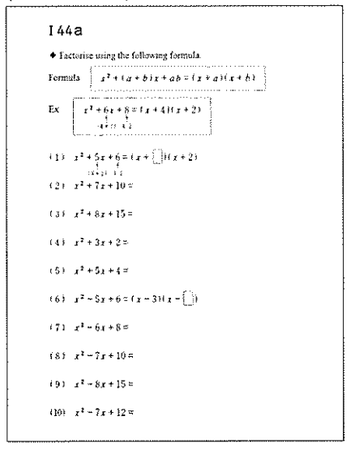
(2) $\begin{cases}7x+2y=12 & \cdots①\\ 5x+2y=8 & \cdots②\end{cases}$

[Sol] ① − ②:
☐ $x=$ ☐
$x=$ ☐

Substituting this into ①,

Ans. $(x, y)=($, $)$

Level I
(Factorisation)


I 44a

◆ Factorise using the following formula.

Formula $x^2+(a+b)x+ab=(x+a)(x+b)$

Ex $x^2+6x+8=(x+4)(x+2)$
$4\times2=8$, $4+2=6$

(1) $x^2+5x+6=(x+$ ☐$)(x+2)$

(2) $x^2+7x+10=$

(3) $x^2+8x+15=$

(4) $x^2+3x+2=$

(5) $x^2+5x+4=$

(6) $x^2-5x+6=(x-3)(x-$ ☐$)$

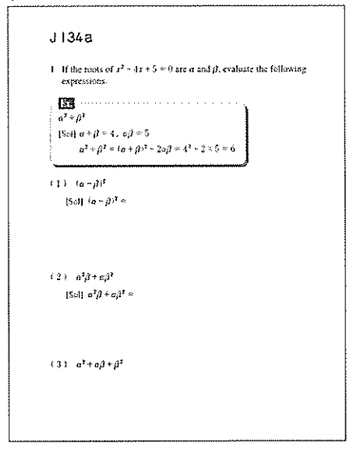
(7) $x^2-6x+8=$

(8) $x^2-7x+10=$

(9) $x^2-8x+15=$

(10) $x^2-7x+12=$

Level J
(Root-Coefficient Relationships)


J I 34a

1 If the roots of $x^2-4x+5=0$ are α and β, evaluate the following expressions.

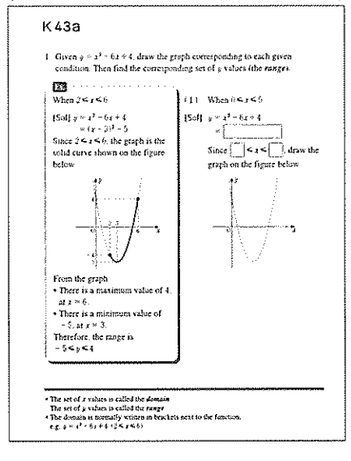
Ex
$\alpha^2+\beta^2$
[Sol] $\alpha+\beta=4$, $\alpha\beta=5$
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=4^2-2\times5=6$

(1) $(\alpha-\beta)^2$
[Sol] $(\alpha-\beta)^2=$

(2) $\alpha^2\beta+\alpha\beta^2$
[Sol] $\alpha^2\beta+\alpha\beta^2=$

(3) $\alpha^3+\alpha\beta+\beta^2$

Level K
(Maxima and Minima of Quadratic Functions)


K 43a

1 Given $y=x^2-6x+4$, draw the graph corresponding to each given condition. Then find the corresponding set of y values (the *range*).

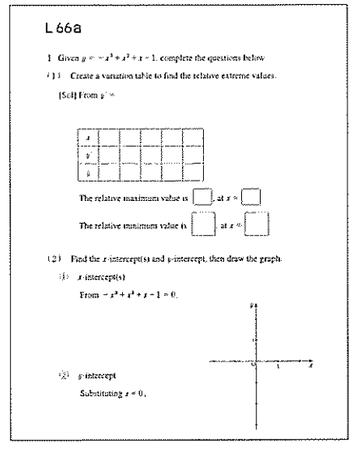
Ex
When $2\le x\le6$
[Sol] $y=x^2-6x+4$
$=(x-3)^2-5$
Since $2\le x\le6$, the graph is the solid curve shown on the figure below.

From the graph
• There is a maximum value of 4, at $x=6$.
• There is a minimum value of -5, at $x=3$.
Therefore, the range is $-5\le y\le4$.

(1) When $0\le x\le5$
[Sol] $y=x^2-6x+4$
$=$ ☐
Since ☐ $\le x\le$ ☐, draw the graph on the figure below.

• The set of x values is called the *domain*.
The set of y values is called the *range*.
• The domain is normally written in brackets next to the function.
e.g. $y=x^2-6x+4$ $(2\le x\le6)$

Level L
(Relative Maxima and Minima)


L 66a

1 Given $y=-x^3+x^2+x-1$, complete the questions below.

(1) Create a variation table to find the relative extreme values.

[Sol] From $y'=$

x						
y'						
y						

The relative maximum value is ☐ at $x=$ ☐

The relative minimum value is ☐ at $x=$ ☐

(2) Find the x-intercept(s) and y-intercept, then draw the graph.

(i) x-intercept(s)
From $-x^3+x^2+x-1=0$,

(ii) y-intercept
Substituting $x=0$,

Printed in Malaysia FM 1805

TABLE OF LEARNING MATERIALS II-2 (Level M to X)

KUMON

SCT: Standard Completion Time (Min./Sheet)
This is the set time in which the student should complete each worksheet, including time taken for correction.

July 2008

	M	SCT	N	SCT	O	SCT
1- 10	Trigonometric Functions I	6–12	Loci I	9–18	Advanced Differentiation I	10–20
11- 20	Trigonometric Functions II	6–12	Loci II	10–20	Advanced Differentiation II	10–20
21- 30	Trigonometric Functions III	8–16	Quadratic Inequalities and Regions	8–16	Increasing and Decreasing Functions	17–34
31- 40	Trigonometric Functions IV	8–16	Arithmetic Sequences	8–16	Concavity and Tangents	17–34
41- 50	Trigonometric Functions V	8–16	Sums of Arithmetic Sequences	8–16	Maxima and Minima	17–34
51- 60	Trigonometric Functions VI	8–16	Geometric Sequences	8–16	Applications of Differential Calculus I	20–40
61- 70	Trigonometric Equations	8–16	Sums of Geometric Sequences	9–18	Applications of Differential Calculus II	20–40
71- 80	Graphs of Trigonometric Functions I	8–16	Various Sequences	10–20	Indefinite Integrals I	10–20
81- 90	Graphs of Trigonometric Functions II	8–16	Infinite Sequences	8–16	Indefinite Integrals II	10–20
91-100	Trigonometric Inequalities	10–20	Infinite Geometric Sequences I	8–16	Indefinite Integrals III	12–24
101-110	Maxima and Minima	10–20	Infinite Geometric Sequences II	9–18	Definite Integrals I	10–20
111-120	Addition Formulas I	6–12	Infinite Geometric Series	10–20	Definite Integrals II	12–24
121-130	Addition Formulas II	8–16	Infinite Series	10–20	Advanced Integration	15–30
131-140	Addition Formulas III	10–20	Limits of Functions I	8–16	Applications of Integrals I	15–30
141-150	Coordinates of a Point	8–16	Limits of Functions II	9–18	Applications of Integrals II	17–34
151-160	Equations of Lines I	8–16	Limits of Trigonometric Functions	10–20	Applications of Integrals III	17–34
161-170	Equations of Lines II	10–20	Continuous and Discontinuous Functions	10–20	Applications of Integrals IV	17–34
171-180	Equations of Lines III	10–20	Differentiation I	8–16	Differential Equations I	14–28
181-190	Equations of Circles	8–16	Differentiation II	9–18	Differential Equations II	17–34
191-200	Tangents of Circles	10–20	Differentiation III	10–20	Differential Equations III	20–40

	XT	SCT
1- 10	Sine and Cosine Theorems	8–16
11- 20	Application I	10–20
21- 30	Application II	12–24

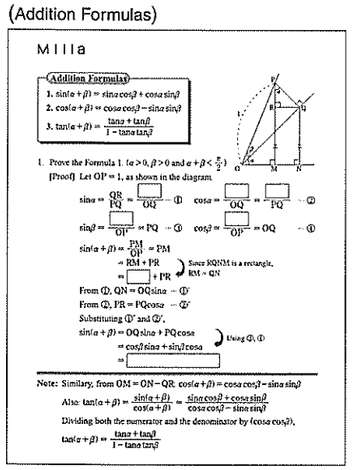
	XV	SCT
1- 10	Surface Vectors I	6–12
11- 20	Surface Vectors II	8–16
21- 30	Surface Vectors III	8–16
31- 40	Surface Vectors IV	10–20
41- 50	Coordinates in Space	6–12
51- 60	Vectors in Space	8–16
61- 70	Basics of Inner Products of Vectors	8–16
71- 80	Inner Products of Vectors	8–16
81- 90	Applications and Summary of Vectors	10–20
91-100	Vectors and Figures	10–20
101-110	Equations of Lines and Planes in Space I	10–20
111-120	Equations of Lines and Planes in Space II	10–20
121-130	Equations of Lines and Planes in Space III	12–24
131-140	Equations of Lines, Planes and Spheres	12–24

	XM	SCT
1- 10	Matrix Definitions, Addition and Subtraction	6–12
11- 20	Matrix Multiplication	8–16
21- 30	Inverse Matrices	10–20
31- 40	Equations and Matrices	10–20
41- 50	Mapping I	8–16
51- 60	Mapping II	10–20
61- 70	Transformations I	8–16
71- 80	Transformations II	10–20
81- 90	Transformations III	10–20

	XP	SCT
1- 10	Introduction to Permutations	6–12
11- 20	Permutations	6–12
21- 30	Permutations and Combinations	8–16
31- 40	Combinations	10–20
41- 50	Binomial Theorem	12–24
51- 60	Probability	8–16
61- 70	Conditional Probability	10–20
71- 80	Independent Trials	12–24
81- 90	Expected Value	12–24

	XS	SCT
1- 10	Introduction to Statistics	8–16
11- 20	Binomial Distribution	8–16
21- 30	Probability Density Function	10–20
31- 40	Standard Normal Distribution I	12–24
41- 50	Standard Normal Distribution II	12–24
51- 60	Sampling	15–30
61- 70	Hypothesis Testing	15–30

Level M
(Addition Formulas)


M111a

Addition Formulas
1. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
3. $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

1. Prove the Formula 1. for $\alpha>0$, $\beta>0$ and $\alpha+\beta<\frac{\pi}{2}$.

[Proof] Let OP = 1, as shown in the diagram.

$\sin\alpha = \frac{QR}{PQ} = \frac{\square}{OQ}$ … ① $\cos\alpha = \frac{\square}{OQ} = \frac{\square}{PQ}$ … ②

$\sin\beta = \frac{\square}{OP} = PQ$ … ③ $\cos\beta = \frac{\square}{OP} = OQ$ … ④

$\sin(\alpha+\beta) = \frac{PM}{OP} = PM$
$= RM + PR$ (Since RQNM is a rectangle, RM = QN)
$= \square + PR$

From ②, $QN = OQ\sin\alpha$ … ⑤'
From ③, $PR = PQ\cos\alpha$ … ⑥'
Substituting ⑤' and ⑥', $\sin(\alpha+\beta) = OQ\sin\alpha + PQ\cos\alpha$ (Using ④, ①)
$= \cos\beta\sin\alpha + \sin\beta\cos\alpha$
$= \square$

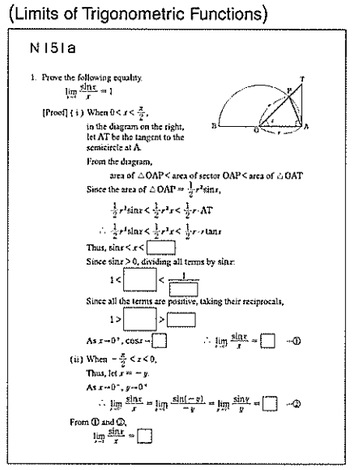
Note: Similarly, from OM = ON − QR, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Also: $\tan(\alpha+\beta) = \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

Dividing both the numerator and the denominator by $(\cos\alpha\cos\beta)$,

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

Level N
(Limits of Trigonometric Functions)


N151a

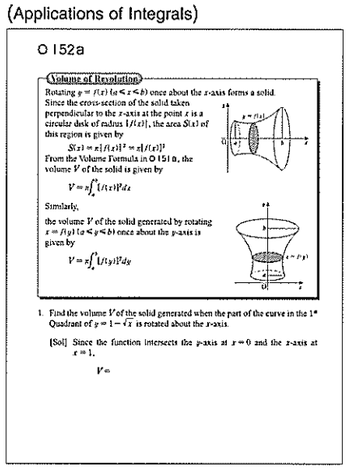
1. Prove the following equality.
$\lim_{x\to+0}\frac{\sin x}{x} = 1$

[Proof] (i) When $0<x<\frac{\pi}{2}$,
in the diagram on the right, let AT be the tangent to the semicircle at A.
From the diagram,
area of △OAP < area of sector OAP < area of △OAT
Since the area of △OAP = $\frac{1}{2}r^2\sin x$,
$\frac{1}{2}r^2\sin x < \frac{1}{2}r^2x < \frac{1}{2}r\cdot AT$
$\therefore \frac{1}{2}r^2\sin x < \frac{1}{2}r^2x < \frac{1}{2}r\cdot r\tan x$
Thus, $\sin x < x < \square$
Since $\sin x > 0$, dividing all terms by $\sin x$:
$1 < \square < \frac{1}{\square}$
Since all the terms are positive, taking their reciprocals,
$1 > \square > \square$
As $x\to+0$, $\cos x \to \square$ $\therefore \lim_{x\to+0}\frac{\sin x}{x} = \square$ … ①

(ii) When $-\frac{\pi}{2}<x<0$,
Thus, let $x = -y$.
As $x\to-0$, $y\to+0$
$\therefore \lim_{x\to-0}\frac{\sin x}{x} = \lim_{y\to+0}\frac{\sin(-y)}{-y} = \lim_{y\to+0}\frac{\sin y}{y} = \square$ … ②

From ① and ②, $\lim_{x\to0}\frac{\sin x}{x} = \square$

Level O
(Applications of Integrals)


O152a

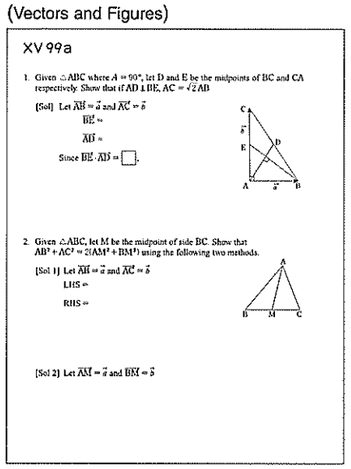
Volume of Revolution
Rotating $y = f(x)$ $(a \le x \le b)$ once about the x-axis forms a solid.
Since the cross-section of the solid taken perpendicular to the x-axis at the point x is a circular disk of radius $|f(x)|$, the area $S(x)$ of this region is given by
$S(x) = \pi|f(x)|^2 = \pi\{f(x)\}^2$
From the Volume Formula in O150, the volume V of the solid is given by
$V = \pi\int_a^b \{f(x)\}^2 dx$

Similarly,
the volume V of the solid generated by rotating $x = f(y)$ $(a \le y \le b)$ once about the y-axis is given by
$V = \pi\int_a^b \{f(y)\}^2 dy$

1. Find the volume V of the solid generated when the part of the curve in the 1st Quadrant of $y = 1 - \sqrt{x}$ is rotated about the x-axis.

[Sol] Since the function intersects the y-axis at $x = 0$ and the x-axis at $x = 1$,
$V =$

Level XV
(Vectors and Figures)


XV99a

1. Given △ABC where $A = 90°$, let D and E be the midpoints of BC and CA respectively. Show that if AD ⊥ BE, $AC = \sqrt{2}AB$

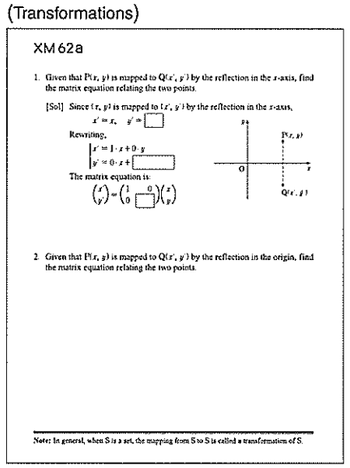
[Sol] Let $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$
$\overrightarrow{BE} =$
$\overrightarrow{AD} =$
Since $\overrightarrow{BE}\cdot\overrightarrow{AD} = \square$,

2. Given △ABC, let M be the midpoint of side BC. Show that $AB^2 + AC^2 = 2(AM^2 + BM^2)$ using the following two methods.

[Sol 1] Let $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$
LHS =
RHS =

[Sol 2] Let $\overrightarrow{AM} = \vec{a}$ and $\overrightarrow{BM} = \vec{b}$

Level XM
(Transformations)


XM62a

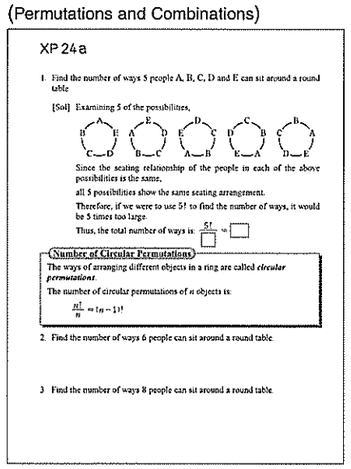
1. Given that P(x, y) is mapped to Q(x′, y′) by the reflection in the x-axis, find the matrix equation relating the two points.

[Sol] Since (x, y) is mapped to (x′, y′) by the reflection in the x-axis,
$x' = x$, $y' = \square$
Rewriting,
$\begin{cases} x' = 1\cdot x + 0\cdot y \\ y' = 0\cdot x + \square \end{cases}$
The matrix equation is:
$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \square \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

2. Given that P(x, y) is mapped to Q(x′, y′) by the reflection in the origin, find the matrix equation relating the two points.

Note: In general, when S is a set, the mapping from S to S is called a transformation of S.

Level XP
(Permutations and Combinations)


XP24a

1. Find the number of ways 5 people A, B, C, D and E can sit around a round table

[Sol] Examining 5 of the possibilities,

Since the seating relationship of the people in each of the above possibilities is the same,
all 5 possibilities show the same seating arrangement.
Therefore, if we were to use 5! to find the number of ways, it would be 5 times too large.
Thus, the total number of ways is $\frac{5!}{\square} = \square$

Number of Circular Permutations
The ways of arranging different objects in a ring are called *circular permutations*.
The number of circular permutations of n objects is:
$\frac{n!}{n} = (n-1)!$

2. Find the number of ways 6 people can sit around a round table.

3. Find the number of ways 8 people can sit around a round table.